The Ministry of Education and Science of the Russian Federation

Lobachevsky State University of Nizhni Novgorod

Computing Mathematics and Cybernetics faculty

The competitiveness enhancement program
of the Lobachevsky State University of Nizhni Novgorod
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Strategic initiative
“Achieving leading positions in the field of supercomputer technology
and high-performance computing”

NUMERICAL METHODS FOR SOLVING DIFFERENTIAL EQUATIONS

Syllabus

Nizhni Novgorod

2014

INTRODUCTION

This course explains numerical methods for solving ordinary and partial differential equations and approaches to their parallelization for shared memory systems.

The course is based on materials developed in UNN with the support of Intel from 2011 through 2013 (http://www.hpcc.unn.ru/?doc=491). In 2014, within the UNN competitive growth program, part of course materials underwent modification. It mostly consisted in a detailed independent work planning. In addition, the key course elements were translated into English.

OBJECTIVES

The main objective of this course is to study numerical methods for solving ordinary and partial differential equations and approaches to their parallelization for shared memory systems.

This involves **the following**:

1. Studying numerical methods for solving systems of ordinary differential equations
2. Studying numerical methods for solving systems of stochastic differential equations
3. Studying numerical methods for solving systems of partial differential equations
4. Studying approaches to checking correctness and convergence of experimental results to theoretical data.
5. Studying principles of parallel algorithm construction for solving differential equations.
6. Mastering parallel programming on shared memory systems (OpenMP, TBB, Cilk Plus).
7. Studying functionality of libraries to solve auxiliary problems such as random number generation and Fourier transform (Intel MKL, FFTW).

REQUIREMENTS TO STUDENTS

The lecture part is intended for graduates familiar with basics of numerical methods to the extent of bachelor courses.

For practice, students are required to have basic C/C++ software development skills and experience in OpenMP parallel programming. TBB, MKL and other Intel Parallel Studio XE capability skills will be useful but are not mandatory. Course materials will provide students will all relevant information about their use.

SYLLABUS

The course duration is 36 hours, i. e. 12 hours of lectures, 8 hours of practice and 16 hours of independent work:

№	Type of class	Name	Time	Practice
1	Lecture	Numerical Methods for Solving Systems of Ordinary Differential Equations	2	1
2	Practice	Numerical Solution of Ordinary Differential Equations as Illustrated by Brain Modeling Problem	2	2
3	Lecture	Numerical Methods for Solving Systems of Stochastic Differential Equations	2	1
4	Practice	Methods for Solving Systems of Stochastic	2	2

		Differential Equations as Illustrated by Financial Market Modeling		
5	Lecture	Solving Partial Differential Equations as Illustrated by Wave Equation and Heat Transfer Equation	4	1
6	Practice	Solving Partial Differential Equations as Illustrated by the Problem of Compound Option Price Computation	2	4
7	Lecture	Solving Partial Differential Equations as Illustrated by the Dirichlet Problem Posed for Poisson's Equation	4	1
8	Practice	Fast Fourier Transform for the Problem of Heat Diffusion in a Plate	2	4
		TOTAL:	20 hours	16 hours

EXPECTED RESULTS

Having completed this course, students will:

- Know numerical methods for solving system of ordinary, stochastic and partial differential equations as well as algorithms for their sequential and parallel implementation.
- Know how to use the studied methods for solution of applied problems, check the numerical results for correctness, analyze their compliance with theory.
- Have parallel application development skills for shared memory systems using OpenMP and TBB, skills related to part of OpenMP and TBB functionality, application efficiency and correctness evaluation skills using Intel Parallel Studio XE tools.

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