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Strategic initiative “Achieving leading positions in the field
of supercomputer technology and high-performance computing”

Parallel Programming
for Multiprocessor Distributed Memory Systems

07 Practice
Parallel Methods of Solving the Linear Equation Systems

Brief description

Nizhni Novgorod
2014

07_PRACTICE. PARALLEL METHODS OF SOLVING THE LINEAR EQUATION SYSTEMS

OBJECTIVES

An objective of the practice is to demonstrate a practical application of the parallel linear algebra algorithms by example of solving the linear equation systems.

ABSTRACT

The work is organized in the following way. The problem of solving the linear equation systems is stated. Gauss algorithm is discussed. Implementation of serial solving method is demonstrated. Possible parallel algorithm and scheme of data distribution are considered. Implementation of parallel algorithm using MPI is described.

BRIEF OVERVIEW

Linear equation systems appear in the course of solving a number of applied problems, which are formulated by differential, integral equations or by systems of non-linear (transcendent) equations. They may appear also in the problems of mathematical programming, statistical data processing, function approximation, or in discretization of boundary differential problems by methods of finite differences or of finite elements, etc.

The coefficient matrices of linear equation systems may be of various structures and have various characteristics. The matrices of the systems solved may be dense and their order may reach several thousands of rows and columns. In solving many problems there can be the systems, which possess symmetric positively definite stripe matrices with the order of tens of thousands and the width of the stripe of the several thousand elements. And finally in consideration of a great number of problems there may appear systems of linear equations with sparse matrices of the order of millions of columns and rows.

This practice discusses one of the direct methods of solving linear equation systems, i.e. the Gauss method and its parallel generalization.

The first section of practice contains the problem statement of solving the linear equation systems.

In the second section of the practice Gauss algorithm is considered including Gauss elimination and back substitution stages.

In the third section the project for Microsoft Visual Studio is developed step-by-step. The developed application implements the serial algorithm as well as the necessary steps to input initial data (matrix and vector), finish the execution correctly, and carry out the computational experiments.

In the next section the data distribution scheme is considered.

The last section is devoted to implementation of previously described parallel algorithm as an MPI parallel program. Serial implementation is used as the basis. Parallel program is developed step-by-step like serial one. Necessary steps include parallel program initialization, data input (matrices), data distribution, parallel Gauss elimination and back substitution, gathering the results.

The parallel variant of the Gauss method is based on the row-wise block-striped matrix distribution among the processors and the use of the cyclic scheme of row distribution. This scheme makes possible to balance the computational load. To develop the parallel variant of the method a complete design cycle is carried out. The basic computational subtasks are defined, information communications are analyzed, the issues of scaling are discussed, efficiency characteristics are estimated, software implementation is suggested and the results of the computational experiments are given. According to the efficiency analysis, the use of the parallel variant of the Gauss method does not provide the speedup of computations, because of the big number of communication operations.

FOR STUDENTS

The description of the classical variants of the algorithms is included in Kincaid, et al. (1991) and Burden, et al. (2000). The implementations of the parallel algorithms on pseudocode (for the case of distributed memory) are stated in Quinn (2004).

REFERENCES

1. Kincaid D.R. and Cheney E.W. Numerical analysis: mathematics of scientific computing. Brooks/Cole Publishing Company, 1991.
2. Burden R.L., Faires J.D. Numerical Analysis. Brooks Cole, 2000.
3. Quinn, M.J. (2004). Parallel Programming in C with MPI and OpenMP. – New York, NY: McGraw-Hill.
4. Kumar V., Grama, A., Gupta, A., Karypis, G. (1994). Introduction to Parallel Computing. - The Benjamin/Cummings Publishing Company, Inc. (2nd edn., 2003)

5. Foster, I. (1995). Designing and Building Parallel Programs: Concepts and Tools for Software Engineering. Reading, MA: Addison-Wesley.

EXERCISES

1. Study the conjugate gradient method of solving the linear equation systems.
2. Develop the serial and the parallel variants of the method.

TEST QUESTIONS

1. What is the complexity order of the Gaussian elimination method when applying to triangular matrix?
 - a. $O(n)$
 - b. (+) $O(n^2)$
 - c. $O(n^3)$
2. What is the complexity order of the Gaussian elimination method when applying to square matrix?
 - a. $O(n)$
 - b. $O(n^2)$
 - c. (+) $O(n^3)$
3. What is the relation between the error of the Gaussian method without choosing the pivot element and the error of the Gaussian method with choosing the pivot element by column?
 - a. The errors are the same and they are comparable to rounding error
 - b. The error of the common Gaussian method is smaller than the error of the method with choosing the pivot element
 - c. (+) The error of the Gaussian method with choosing the pivot element is smaller than of the common method.
4. What is the relation between the error of the Gaussian method without choosing the pivot element and the error of the Gaussian method with choosing the pivot element by column and by row?
 - a. The errors are the same and they are comparable to rounding error
 - b. The error of the common Gaussian method is smaller than of the method with choosing the pivot element
 - c. (+) The error of the Gaussian method with choosing the pivot element is smaller than of the common method.

5. What number of processes may be used during the execution of parallel implementation of Gaussian method?
 - a. (+) Any possible number of processes may be used.
 - b. The number of processes should be a perfect square.
 - c. The number of processes should be equal the number of matrix rows.
6. What function should be used to distribute between processes the matrix size in parallel Gaussian method?
 - a. MPI_Send
 - b. (+) MPI_Bcast
 - c. MPI_Scatter
 - d. MPI_Gather
7. What function should be used to distribute between processes the matrix in parallel Gaussian method?
 - a. MPI_Send
 - b. MPI_Bcast
 - c. (+) MPI_Scatter
 - d. MPI_Gather
8. What function should be used to get the result vector in parallel Gaussian method (Gaussian elimination stage)?
 - a. (+) MPI_AllReduce
 - b. MPI_Bcast
 - c. MPI_Scatter
 - d. MPI_Gather
9. What function should be used to get the result vector in parallel Gaussian method (back substitution stage)?
 - a. MPI_AllReduce
 - b. MPI_Bcast
 - c. MPI_Scatter
 - d. (+) MPI_Gather
10. What virtual topology should be used to implement parallel Gaussian method?
 - a. (+) MPI_COMM_WORLD is enough.
 - b. Cartesian topology
 - c. Special graph topology