


Lobachevsky State University of Nizhni Novgorod

Computing Mathematics and Cybernetics faculty

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Lobachevsky State University of Nizhni Novgorod

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Parallel Programming for Multiprocessor Distributed Memory Systems

08 Practice
Parallel Algorithms of Graph Processing

With the support of Microsoft

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Software department

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- The Shortest Path Problem Statement
- Serial Floyd Algorithm Implementation
- Parallel Floyd Algorithm
- Parallel Floyd Program

THE SHORTEST PATH PROBLEM STATEMENT

Step 1. Problem Statement…

- Let *G* be a graph

$$G = (V, R),$$

V – the set of vertices

$$v_i, 1 \le i \le n$$

R – the set of arcs

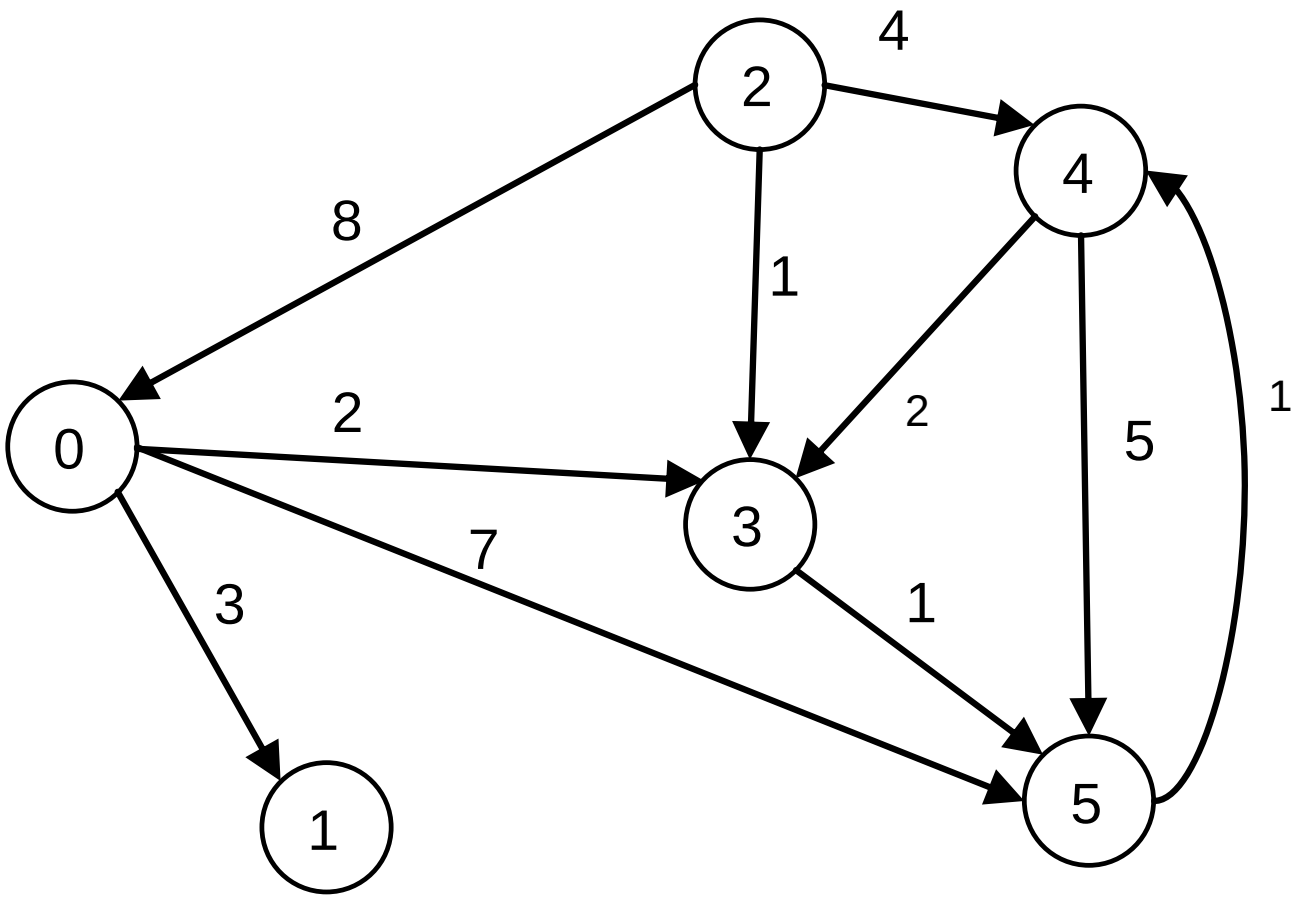
$$r_j = (v_{s_j}, v_{t_j}), 1 \le j \le m$$

- Generally, the arcs may be assigned certain numerical characteristics (weights)

$$w_j, 1 \le j \le m$$

Step 1. Problem Statement…

- Example of the Weighted Oriented Graph



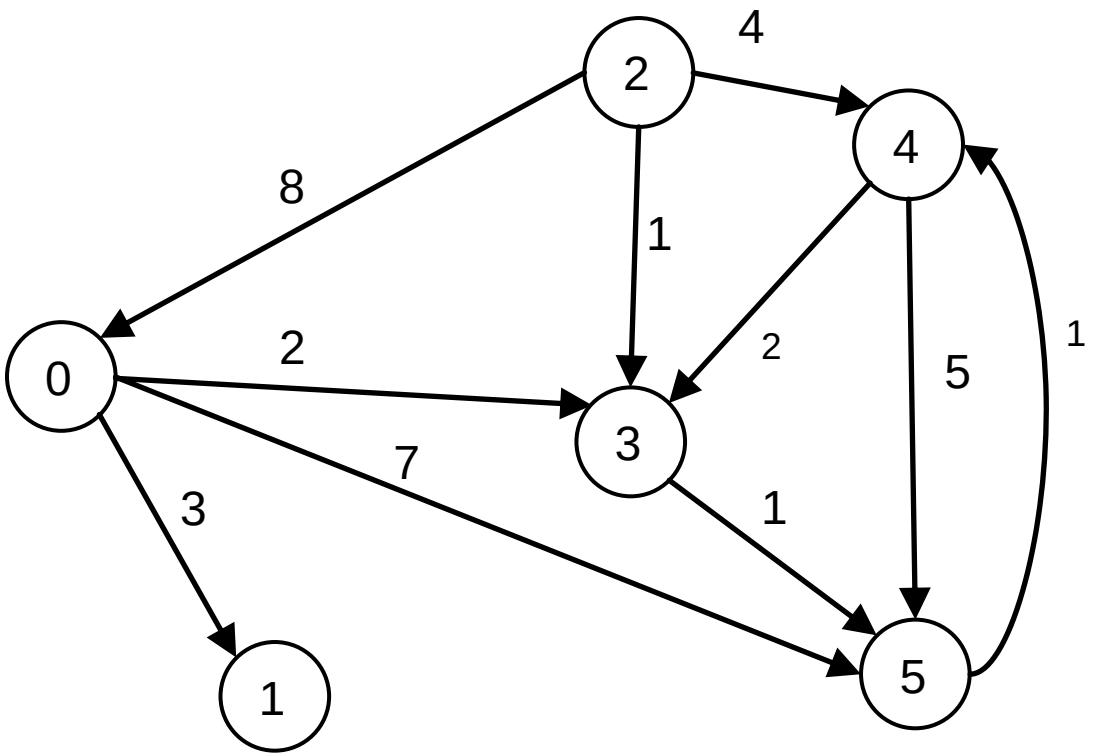
Step 1. Problem Statement…

- Presenting dense graphs, almost all the nodes of which are linked by arcs (i.e. $m \approx n^2$), may be efficiently described by means of the adjacency matrix

$$A = (a_{ij}),\ 1 \le i, j \le n, \qquad a_{ij} = \begin{cases} w(v_i, v_j), & \text{если } (v_i, v_j) \in R, \\ 0, & \text{если } i = j, \\ \infty, & \text{иначе}. \end{cases}$$

Step 1. Problem Statement…

- The Adjacency Matrix



$$\begin{pmatrix} 0 & 3 & \infty & 2 & \infty & 7 \\ \infty & 0 & \infty & \infty & \infty & \infty \\ 8 & \infty & 0 & 1 & 4 & \infty \\ \infty & \infty & \infty & 0 & \infty & 1 \\ \infty & \infty & \infty & 2 & 0 & 5 \\ \infty & \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

Step 1. Problem Statement…

The problem of searching the shortest paths

- The initial information for the problem is the weighted graph
- Each arc of the graph is assigned non-negative weight
- The graph is assumed to be oriented
- The problem is to find out the shortest paths among all the pairs of destination vertices for the given graph *G*
- As a method for solving the problem, we will further use Floyd algorithm

Step 1. Problem Statement

- Generally, the algorithm may be presented in the following way

```
// Serial Floyd algorithm
for (k = 0; k < n; k++)
  for (i = 0; i < n; i++)
    for (j = 0; j < n; j++)
      A[i,j] = min(A[i,j],A[i,k]+A[k,j]);
```

- The complexity of the Floyd algorithm is of n^3 order

SERIAL FLOYD ALGORITHM IMPLEMENTATION

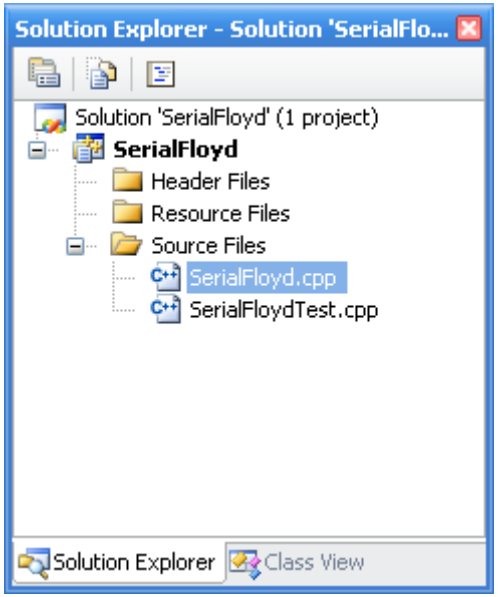
Step 2. Serial Implementation…

- Task 1 – Open the project SerialFloyd
- Task 2 – Input the Number of Vertices
- Task 3 – Terminate the Program Execution
- Task 4 – Implement the Floyd Algorithm
- Task 5 – Carry out the Computational Experiments

Step 2. Serial Implementation…

Task 1 – Open the project SerialFloyd

- Start **Microsoft Visual Studio**
- Open solution ParallelGauss.sln from the folder **c:\ParLabs\SerialFloyd**
- Open file **SerialFloyd.cpp** in the window Solution Explorer (Ctrl+Alt+L)



Step 2. Serial Implementation…

Task 2 – Input the Number of Vertices

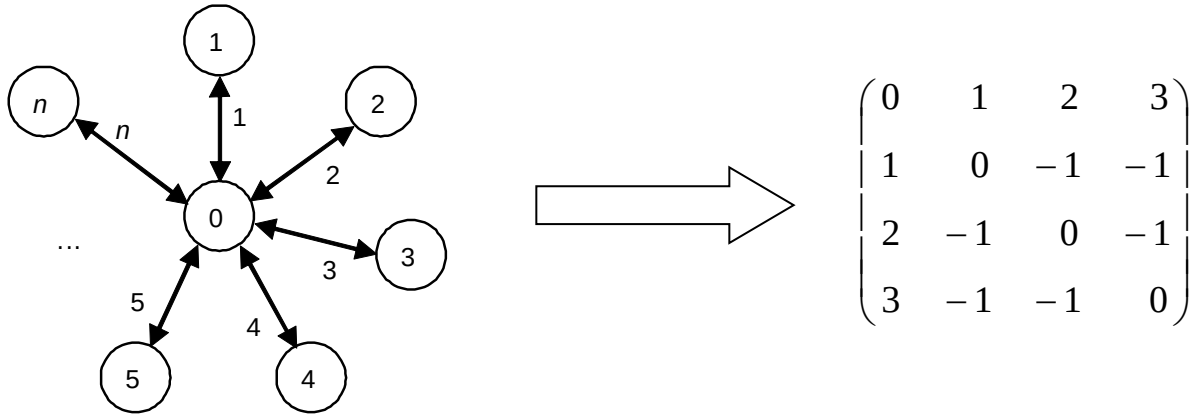
- To set the initial data of the serial Floyd algorithm develop the function **`ProcessInitialization()`**
- This function is intended for
  - the initialization of all the variables used in the program
  - the input of the number of the processed vertices (the adjacency matrix size)
  - the allocation of the memory for the adjacency matrix
  - filling the memory with the initial values

```
// Function for process initialization
void ProcessInitialiazation(int *&pMatrix, int& Size);
```

Step 2. Serial Implementation…

Task 2 – Input the Number of Vertices

- Implement the functions **`DummyDataInitialization()`**
- Use the simple method of implementation, containing the set of data, the correctness of which can be easily checked



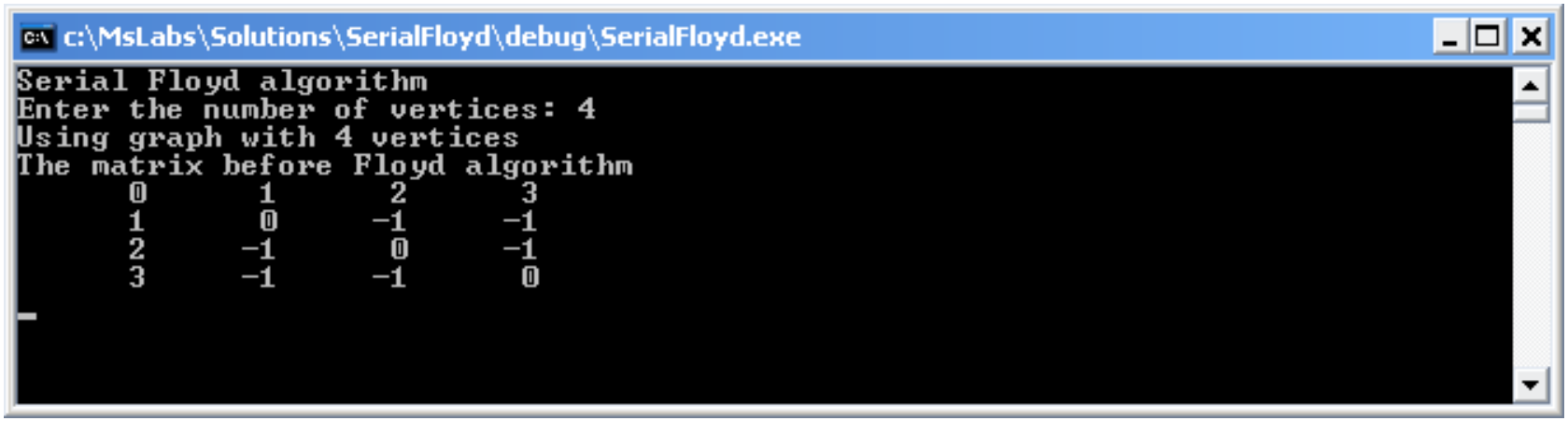
$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & -1 & -1 \\ 2 & -1 & 0 & -1 \\ 3 & -1 & -1 & 0 \end{pmatrix}$$

```
// Function for simple setting the initial data
void DummyDataInitialization(int *pMatrix, int Size);
```

Step 2. Serial Implementation…

Task 2 – Input the Number of Vertices

- Develop the function **`ProcessInitialization()`**, **`DummyDataInitialization()`** and **`PrintMatrix()`**
- Call the functions **`ProcessInitialization()`** and **`PrintMatrix()`** from main function of the application
- Compile and run the application



Step 2. Serial Implementation…

Task 3 –Terminate the Program Execution

- The function for correct program termination **`ProcessTermination()`**

```
// Function for computational process termination
void ProcessTermination(int *pMatrix);
```

- The function **`ProcessTermination()`** should be called at the end of the function **`main()`**

Step 2. Serial Implementation…

Task 4 – Implement the Floyd Algorithm

- To execute the Floyd algorithm develop the function **`SerialFloyd()`**

```
// Serial Floyd algorithm
void SerialFloyd(int *pMatrix, int Size);
```

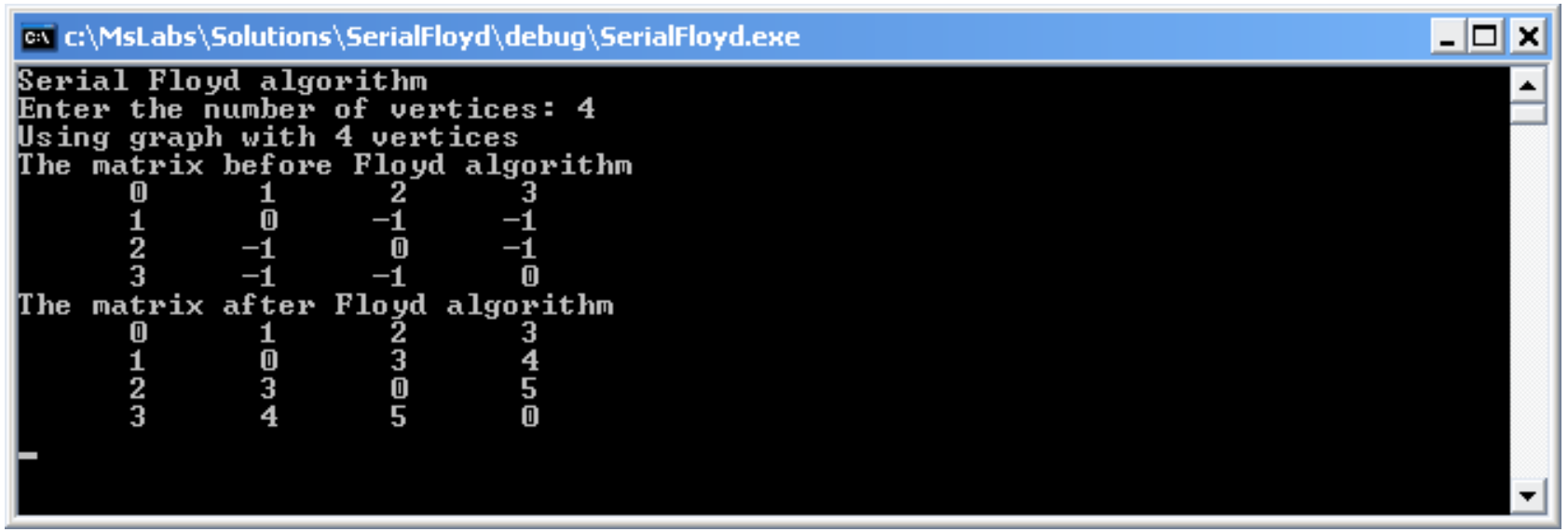
- The value -1 has been chosen to denote the infinitely. Implement the function **`Min()`**

```
int Min(int A, int B) {
  int Result = (A < B) ? A : B;
  if((A < 0) && (B >= 0)) Result = B;
  if((B < 0) && (A >= 0)) Result = A;
  if((A < 0) && (B < 0)) Result = -1;
  return Result;
}
```

Step 2. Serial Implementation…

Task 4 – Implement the Floyd Algorithm

- Implement the function **`SerialFloyd()`**
- Add debugging print to the function **`main()`**
- Compile and run the application



Step 2. Serial Implementation

Task 5 – Carry out the Computational Experiments

- Develop the function **`RandomDataInitialization()`** for setting the data with random values (initialize the random generator by the current time value)

```
// Function for random generating the initial data
void RandomDataInitialization(int *pMatrix, int Size);
```

- Call this function instead of the function **`DummyDataInitialization()`**
- Add time measurement and printing
- Carry out the computational experiments with large objects
- Fill the table with results of experiments

PARALLEL FLOYD ALGORITHM

Step 3. Parallel Algorithm…

Subtask definition

- The effective way of parallel scheme of the Floyd algorithm is to update the values of matrix A simultaneously

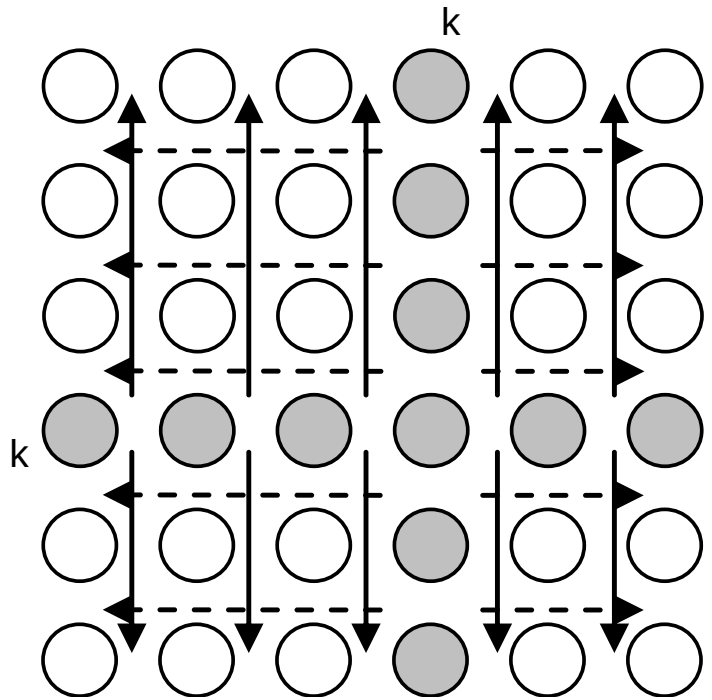
Analysis of Information Dependencies

- Computations in the subtasks become possible only if each subtask (i, j) contains the elements A_{ij}, A_{ik}, A_{kj} of the matrix A
- Each element A_{kj} of the row k of the matrix A must be transmitted to all the subtasks (k, j), $1 \le j \le n$, and each element A_{ik} of the column k of the matrix A must be transmitted to all the subtasks (i, k), $1 \le i \le n$

Step 3. Parallel Algorithm…

Analysis of Information Dependencies

- The Information Dependencies of the Basic Computational Subtasks (the arrows show the direction of exchanging values at iteration *k*)



Step 3. Parallel Algorithm…

Scaling and Distributing the Subtask among the Processors

- A possible way to aggregate the computations is to use of block-striped scheme of the matrix *A* partitioning
- This approach corresponds to uniting in one basic subtask the computations connected with updating the elements of one or several rows (horizontal partitioning) or columns (vertical partitioning) of matrix *A*
- We will further analyze only partitioning the matrix *A* into **horizontal stripes**

PARALLEL FLOYD PROGRAM

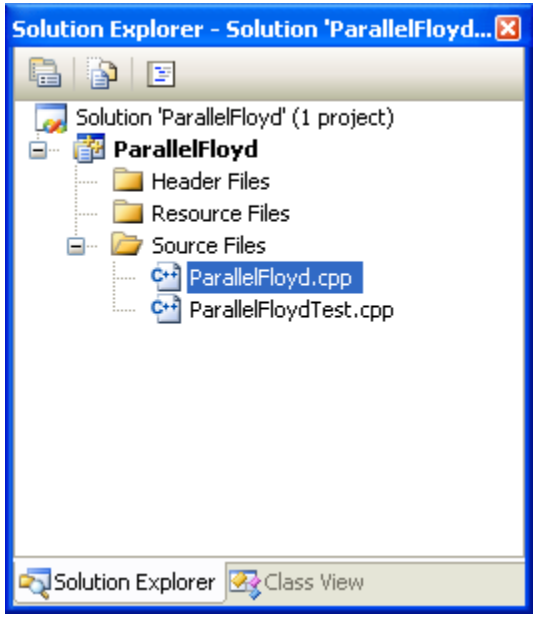
Step 4. Parallel Program…

- Task 1 – Open the Project ParallelFloyd
- Task 2 – Initialize and Finalize the Parallel Program
- Task 3 – Input the Initial Data
- Task 4 – Terminate the Calculations
- Task 5 – Distribute the Data among the Processes
- Task 6 – Implement the Parallel Floyd Algorithm
- Task 7 – Implement the Floyd Algorithm Iterations
- Task 8 – Collect the Result Matrix
- Task 9 – Test the Parallel Program Correctness
- Task 10 – Carry out the Computational Experiments

Step 4. Parallel Program…

Task 1 – Open the Project ParallelFloyd

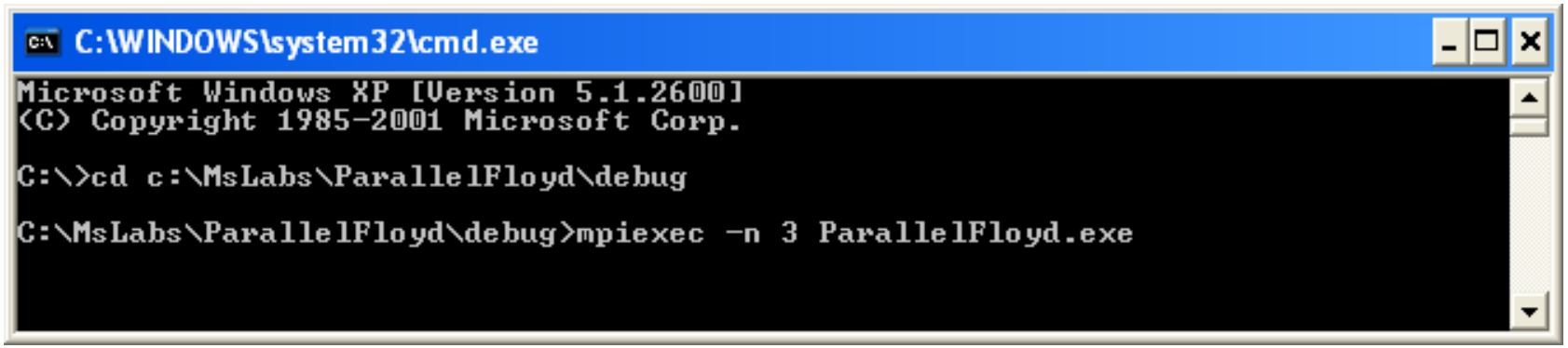
- Start **Microsoft Visual Studio**
- Open solution ParallelFloyd.sln from the folder **c:\ParLabs\ParallelFloyd**
- Open file **ParallelFloyd.cpp** in the window Solution Explorer (Ctrl+Alt+L)



Step 4. Parallel Program…

Task 2 – Initialize and Finalize the Parallel Program

- Initialize the environment of the MPI program execution in the main function
- Determine the number of processes available for MPI program
- Determine the process rank in communicator **`MPI_COMM_WORLD`**
- Set global variables for storing these values (**`ProcNum`** and **`ProcRank`** correspondingly)
- Compile and run the parallel application



Step 4. Parallel Program…

Task 3 – Input the Initial Data

- To initialize the computations develop the function ProcessInitialization
  - input the amount of the number of vertices in the graph
  - broadcast the number of vertices to the other processes
  - allocate memory for the adjacency matrix and the stripes assigned to the processes

```
// Function for process initialization
void ProcessInitialiazation(int *&pMatrix,int *&pProcRows,
  int& Size, int& RowNum);
```

- Implement the function **`ProcessInitialization()`**
- Call the function from the main function of application
- Compile and run the application

Step 4. Parallel Program…

Task 4 –Terminate the Calculations

- Develop the function for correct program termination **`ProcessTermination()`**
- Deallocate the memory for storing the adjacency matrix **`pMatrix`** (on the root process), and the memory for storing the matrix stripes **`pProcRows`**

```
// Function for computational process termination
void ProcessTermination(int *pMatrix, int *pProcRows) {
  if(ProcRank == 0)
    delete [] pMatrix;
  delete [] pProcRows;
}
```

- Call the function from the main function of application
- Compile and run the application

Step 4. Parallel Program…

Task 5 – Distribute the Data among the Processes

- In accordance with the parallel computation scheme the adjacency matrix must be distributed among the processes in equal stripes
- To distribute the matrix **`pMatrix`** use the function **`MPI_Scatterv()`**
- Implement the function **`DataDistribution()`**

```
// Data distribution among the processes
void DataDistribution(int *pMatrix, int *pProcRows,
  int Size, int RowNum);
```

Step 4. Parallel Program…

Task 5 – Distribute the Data among the Processes

- Call the function **`DataDistribution()`** from the main program
- To test the correctness of the data distribution among the processes implement the “debugging print” function **`TestDistribution()`**
  - Print the adjacency matrix **`pMatrix`** on the root process
  - Print the matrix stripes, which are distributed on each of the processes

```
// Data distribution among the processes
void TestDistribution(int *pMatrix, int *pProcRows,
  int Size, int RowNum);
```

Step 4. Parallel Program…

Task 5 – Distribute the Data among the Processes

- Make sure that the data is distributed correctly

```
C:\WINDOWS\system32\cmd.exe
C:\MsLabs\ParallelFloyd\debug>mpiexec -n 3 ParallelFloyd.exe
Parallel Floyd algorithm
Enter the number of vertices: 6
Using graph with 6 vertices
Initial adjacency matrix:
      0      1      2      3      4      5
      1      0     -1     -1     -1     -1
      2     -1      0     -1     -1     -1
      3     -1     -1      0     -1     -1
      4     -1     -1     -1      0     -1
      5     -1     -1     -1     -1      0
ProcRank = 0
Proc rows:
      0      1      2      3      4      5
      1      0     -1     -1     -1     -1
ProcRank = 1
Proc rows:
      2     -1      0     -1     -1     -1
      3     -1     -1      0     -1     -1
ProcRank = 2
Proc rows:
      4     -1     -1     -1      0     -1
      5     -1     -1     -1     -1      0

C:\MsLabs\ParallelFloyd\debug>_
```

Step 4. Parallel Program…

Task 6 – Implement the Parallel Floyd Algorithm

- In accordance with the general scheme of the parallel Floyd algorithm it is necessary to carry out **`Size`** times the operation, which updates the adjacency matrix
  - All the processes need the matrix row, the number of which coincides with the iteration number
  - It is necessary to broadcast this row among the processes
  - Implement the function **`RowDistribution()`**
  - It must use the row number *k* in order to find the process, to which the *k*-th adjacency matrix row belongs, and broadcast the row to the other processes

```
// Function for row broadcasting
void RowDistribution(int *pProcRows, int Size, int RowNum,
  int k, int *pRow);
```

Step 4. Parallel Program…

Task 6 – Implement the Parallel Floyd Algorithm

- The code for the parallel Floyd algorithm will look the following way at the first stage

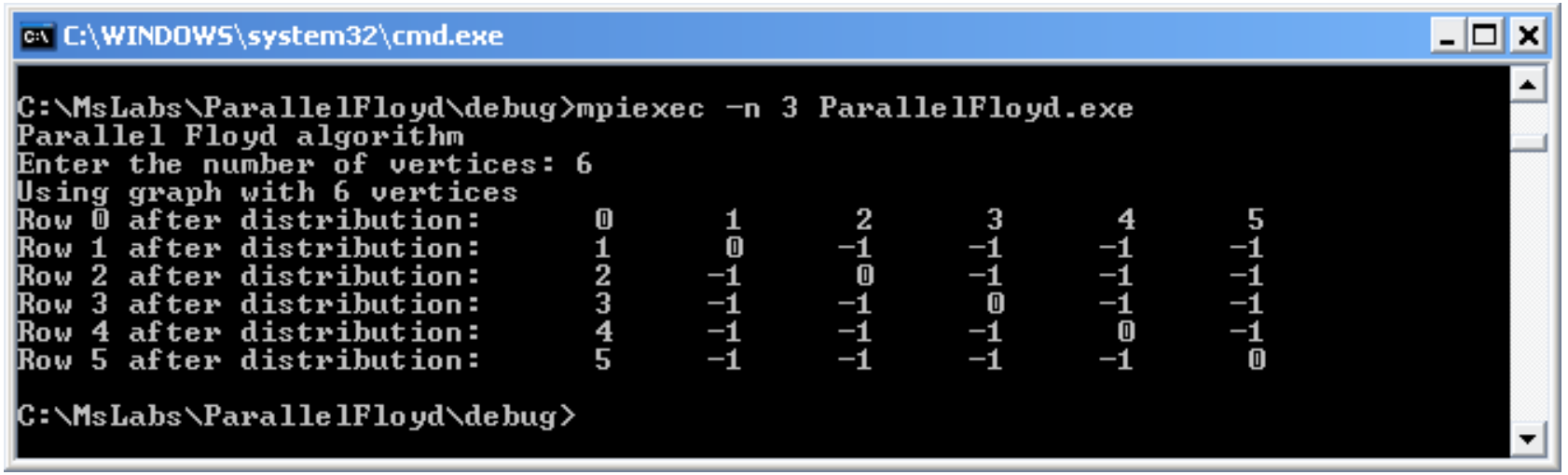
```
// Function for the parallel Floyd algorithm
void ParallelFloyd(int *pProcRows, int Size, int RowNum) {
  int *pRow = new int[Size];

  for (int k = 0; k < Size; k++) {
    // Distribute row among all processes
    RowDistribution(pProcRows, Size, RowNum, k, pRow);
  }
  delete [] pRow;
}
```

Step 4. Parallel Program…

Task 6 – Implement the Parallel Floyd Algorithm

- Implement the function **`RowDistribution()`**
- Call the function **`ParallelFloyd()`** from the main function of the application
- Compile and run the application
- Make sure that the data rows are distributed correctly



Step 4. Parallel Program…

Task 7 – Implement the Floyd Algorithm Iterations

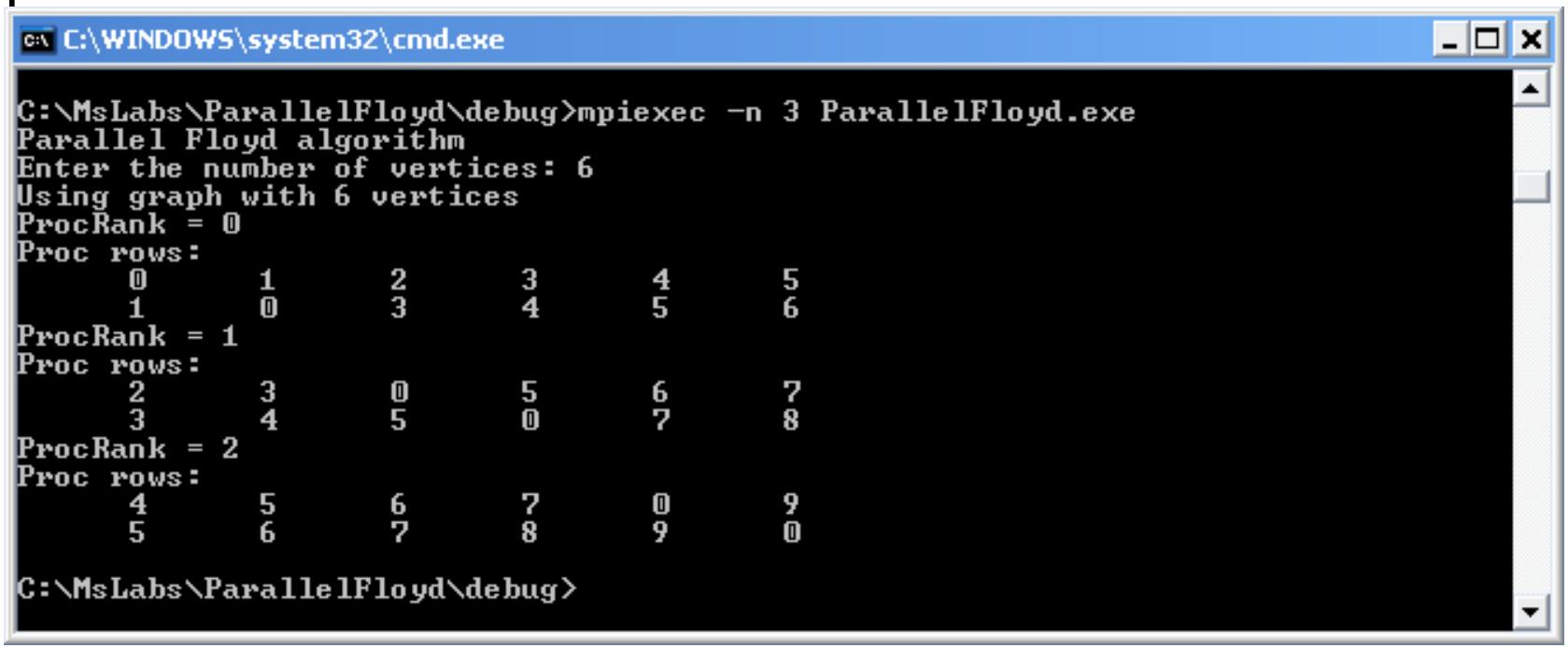
- It is necessary to execute the adjacency matrix update after broadcasting the next adjacency matrix row among the processes
- Add to the function **`ParallelFloyd()`** the following code on each iteration

```
// Update adjacency matrix elements
for (int i = 0; i < RowNum; i++)
  for (int j = 0; j < Size; j++)
    if ((pProcRows[i * Size + k] != -1) &&
      (pRow [j] != -1)) {
      t1 = pProcRows[i * Size + j];
      t2 = pProcRows[i * Size + k] + pRow[j];
      pProcRows[i * Size + j] = Min(t1, t2);
    }
```

Step 4. Parallel Program…

Task 7 – Implement the Floyd Algorithm Iterations

- Compile and run the application
- Check the correctness of the obtained partial results setting different number of processes and different number of the test graph vertices



Step 4. Parallel Program…

Task 8 – Collect the Result Matrix

- To collect the obtained matrix on the root process implement the function **`ResultCollection()`**

```
// Function for process result collection
void ResultCollection(int *pMatrix, int *pProcRows,
  int Size, int RowNum);
```

- Call the function from main function of the application
- Add print of the obtained matrix by means of the function **`PrintMatrix()`** on the process with rank 0
- Compile and run the application
- Check the correctness of the program execution

Step 4. Parallel Program…

Task 9 – Test the Parallel Program Correctness

- To test the correctness of the program develop the function **`TestResult()`**

```
// Testing the result of parallel Floyd algorithm
void TestResult(int *pMatrix, int *pSerialMatrix, int Size);
```

- The function should compare the results of the serial program to the results of the parallel one
- To execute the serial algorithm use the function **`SeriaFloyd()`**
- To make the serial algorithm **`SerialFloyd()`** operate the same data as the developed parallel algorithm **`ParallelFloyd()`**, produce a copy of the data

Step 4. Parallel Program…

Task 9 – Test the Parallel Program Correctness

- Implement the function **`TestResult()`**
- Call the function from main function of the application
- Instead of the function **`DummyDataInitialization()`**, call the function **`RandomDataInitialization()`**
- Compile and run the application
- Set various amounts of the initial data
- Make sure that the application is functioning properly

Step 4. Parallel Program

Task 10 – Carry out the Computational Experiments

- Determine the parallel algorithm execution time
- Carry out the computational experiments with large objects
- Determine the given speedup
- Fill the table with results of experiments

Summary

- Method of solving the shortest path problem (Floyd algorithm) is considered
- Serial and parallel versions of Floyd algorithm are implemented
- Computational experiments are performed, comparison of serial and parallel algorithms is made

Exercises

- Study other parallel algorithms of graph processing:
  - the Prim algorithm for finding the minimum spanning tree
  - the Dejkstra method for solving the problem of finding the shortest path from one of the graph vertices to the other
- Develop the programs, which implement these algorithms

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