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**Introduction to GPU programming**

*Practice 10. Monte Carlo integration and option pricing*

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**Author: S.I. Bastrakov**

# Objectives

The objective of this practice is to apply CURAND for Monte Carlo method applied to two problems: integration and option pricing.

# Abstract

This practice considers two important applications of Monte Carlo method: integration and option pricing. We demonstrate how to use optimized CURAND library via both Host and Device API to solve these problems.

# BRIEF OVERVIEW

We consider pricing of Call European options. A price follows a stochastic model. Monte Carlo simulation is widely used for such simulations. It is performed as follows. Generate a large number of pseudorandom numbers. Computation for each specific number is called Monte Carlo path. For each path we compute price. Result is average of prices of all paths.

Let 𝑟 be pseudorandom number of standard normal distribution. Price of an option is computed as:

If computation is performed with numbers , the result is:

To perform such simulation we need pseudorandom numbers of standard normal distribution. Those can be obtained from numbers uniformly distributed on [0, 1] using the second Box-Muller transform. Let , be numbers on and . Then the following numbers are normally distributed with mean 0 and variance 1:

In the case we consider the fair price can be analytically computed by Black-Scholes formula. We compare result of Monte-Carlo simulation with the analytical result. It must converge with order 1/2, error of Monte Carlo simulation is proportional to .

Implementation on CPU is as follows:

double endCallValue(double s, double x, double r, double mu, double w)

{

double callValue = s \* exp(mu + w \* r) - x;

return (callValue > 0) ? callValue : 0;

}

double MonteCarloCPU(int n, double s, double x, double t, double r, double v)

{

const double mu = (r - 0.5 \* v \* v) \* t;

const double w = v \* sqrt(t);

double \* rnd = new double[n + 1];

srand(12345);

for (int i = 0; i < n; i += 2) {

double u1;

do {

u1 = rand() / (double)RAND\_MAX;

} while (u1 == 0);

double u2 = rand() / (double)RAND\_MAX;

rnd[i] = sqrt(-2.0 \* log(u1)) \* cos(2.0 \* M\_PI \* u2);

rnd[i + 1] = sqrt(-2.0 \* log(u1)) \* sin(2.0 \* M\_PI \* u2);

}

double sum = 0;

for (int i = 0; i < n; i++)

{

double callValue = endCallValue(s, x, rnd[i], mu, w);

sum += callValue;

}

delete [] rnd;

return exp(-r \* t) \* sum / (double)n;

}

Implementation on GPU is left for individual work. We consider 3 versions: naive, using CURAND external, using CURAND internal.

GPU implementation of Monte-Carlo integral using external CURAND:

\_\_global\_\_ void kernel\_curand\_external(int n, float a, float b, float c, float d,

float \* x01, float \* y01, char \* count)

{

int i = blockIdx.x \* blockDim.x + threadIdx.x;

if (i >= n)

return;

if (y01[i] \* (d - c) + c <= func\_device(x01[i] \* (b - a) + a))

count[i] = 1;

else

count[i] = 0;

}

float integrateMonteCarlo\_gpu\_curand\_external(int n, float a, float b, float c,

float d, float \* x01\_device, float \* y01\_device,

char \* count\_host, char \* count\_device)

{

curandGenerator\_t gen;

curandCreateGenerator(&gen, CURAND\_RNG\_PSEUDO\_XORWOW);

curandGenerateUniform(gen, x01\_device, n);

curandGenerateUniform(gen, y01\_device, n);

curandDestroyGenerator(gen);

const int num\_threads\_per\_block = 256;

int num\_blocks = (n + num\_threads\_per\_block - 1) / num\_threads\_per\_block;

kernel\_curand\_external<<<num\_blocks, num\_threads\_per\_block>>>(n, a, b, c, d,

x01\_device, y01\_device, count\_device);

cudaThreadSynchronize();

cudaMemcpy(count\_host, count\_device, n \* sizeof(char),

cudaMemcpyDeviceToHost);

int count = 0;

for (int i = 0; i < n; ++i)

if (count\_host[i])

++count;

return (float)count / (float)n;

}

GPU implementation of Monte-Carlo integral using internal CURAND:

\_\_global\_\_ void kernel\_curand\_internal(int n, float a, float b, float c, float d, char \* count) {

int i = blockIdx.x \* blockDim.x + threadIdx.x;

if (i >= n) return;

curandStateXORWOW\_t state;

curand\_init(2 \* i, 0, 0, &state);

float x = curand\_uniform(&state) \* (b - a) + a;

float y = curand\_uniform(&state) \* (d - c) + c;

if (y <= func\_device(x)) count[i] = 1;

else count[i] = 0;

}

float integrateMonteCarlo\_gpu\_curand\_internal(int n, float a, float b, float c, float d, char \* count\_host, char \* count\_device)

{

const int num\_threads\_per\_block = 256;

int num\_blocks = (n + num\_threads\_per\_block - 1) / num\_threads\_per\_block;

kernel\_curand\_internal<<<num\_blocks, num\_threads\_per\_block>>>(n, a, b, c, d,

count\_device);

cudaThreadSynchronize();

cudaMemcpy(count\_host, count\_device, n \* sizeof(char),

cudaMemcpyDeviceToHost);

int count = 0;

for (int i = 0; i < n; ++i)

if (count\_host[i])

++count;

return (float)count / (float)n;

}

# FOR STUDENTS

Detailed information about CURAND is presented in [1].

# References

1. NVIDIA CURAND Documentation [http://docs.nvidia.com/curand/index.html#axzz3JRcPurfI].

# Individual work

1. Generalize our Monte Carlo integration for integrals of arbitrary dimension.
2. Implement three versions of Monte Carlo option pricing on GPU: naive, using CURAND external, using CURAND internal. Compare performance of these versions.
3. Modify implementation to compute prices of several options with different parameters.
4. Create multi-GPU implementation for multi-option case, each GPU handles one or several options.